

## P.R.GOV.T.COLLEGE (AUTONOMOUS), KAKINADA

### III B.Sc. MATHEMATICS-SEMESTER-VI, PAPER-III

#### Core: Multiple Integrals and Vector Calculus

Total Hrs. of Teaching-Learning: 45 @ 3 h / Week

Total Credits : 03

#### Objectives:

- Introducing the concepts of curve Integrals, Surface Integrals and Volume Integrals.
- Awareness of the concepts of the transformation between curl Integration, Surface Integration and Volume integration.
- Introducing the concepts of geometrical meaning of Gradient, Divergence and Curl.

#### MODULE-I:

##### **MULTIPLE INTEGRALS**

**UNIT:1.** Line Integrals- Definitions-Plane curve, closed curve, simple curve, Jordan curve, length of a polygon inscribed in a curve, rectifiable curve, length of a curve, arc of a curve, functions of bounded variation, line integral-Properties and evaluation of line integrals (only problems )  
(10hours)

**UNIT:2.** Double integral- Evaluation of double integrals, change of order of integration. Surface areas-Definition and evaluation of surface integrals.  
(12 hours)

#### MODULE II :

##### **VECTOR CALCULUS**

**UNIT:3.** Vector differentiation –Ordinary Derivatives of Vector valued functions, Continuity and Differentiation, Gradient , Divergence and Curl.  
(12 hours)

**UNIT:4.** Vector integration – Ordinary integrals of Vector Valued Functions. Green's Theorem in a plane . Divergence Theorem of Gauss, Stokes theorem and applications.  
(11 hours)

#### Prescribed text Book:

A text book of Mathematics, Vol. III, S. Chand & Co.

#### Books for Reference:

1. Dr.B.Leela Lakshmi Kumari, Prof.G. ChakradharaRao, prof. U. Ram Mohan Rao, Prof. N.Bhaskar Reddy, Third year Mathematics Linear Algebra and Vector Calculus, Telugu academy, Hyd.
2. Murray & R. Spiegel, Vector Analysis, Schaum series Publishing Company. ,
3. SanthiNarayana & P.K Mittal, A Course of Mathematical Analysis.

**QUESTION PAPER PATTERN, Semester-VI**

| MODULE           | TOPIC                  | V.S.A.Q   | S.A.Q(including choice) | E.Q(including choice) | Total Marks |
|------------------|------------------------|-----------|-------------------------|-----------------------|-------------|
| <b>MODULE-I</b>  | Line integral          | 02        | 02                      | 01                    | 20          |
|                  | Double integral        | 02        | 03                      | 02                    | 33          |
| <b>MODULE-II</b> | Vector differentiation | 02        | 02                      | 01                    | 20          |
|                  | Vector integration     | 02        | 03                      | 02                    | 33          |
| <b>TOTAL</b>     |                        | <b>08</b> | <b>10</b>               | <b>06</b>             |             |

E.Q = Essay questions ( 8 marks)

S.A.Q = Short answer questions (5 marks)

V.S.A.Q = Very Short answer questions ( 1 mark)

Essay questions :  $4 \times 8M = 32$

Short answer questions :  $5 \times 6M = 30$

Very Short answer questions :  $8 \times 1M = 08$

Total Marks: 70

**P.R GOVT.COLLEGE (AUTONOMOUS), KAKINADA**  
**III B.Sc. MATHEMATICS- SEMESTER-VI,**  
**MODEL PAPER – PAPER-III(Applied)**  
**COURSE-Multiple Integrals & Vector Calculus**  
**(w.e.f.2016-2017)**

Time: 3 hours

Max. marks : 70M

**PART -I**

Answer all the following questions.

8x1M =8M

1. Write the parametric equations of  $y^2=4ax$ .
2. Define the norm of the partition.
3. State the Fubini's theorem.
4. Sketch the area enclosed by  $y=x$  and  $xy=1$ .
5. Find  $\text{div } f$ , where  $f = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$
6. Evaluate  $\int_0^1 (e^t \bar{i} + e^{-2t} \bar{j}) dt$
7. State Green's theorem
8. Write Green's Identities

**PART -II**

Answer any THREE questions from each section.

6x5M =30M

**SECTION - A**

9. Evaluate  $\int_C \frac{dx}{x+y}$  over the curve  $x = at^2$ ,  $y = 2at$  when  $1 \leq t \leq 2$
10. Evaluate  $\int_C x^2 y^2 dx + y dy$  and C is a parabola  $y^2 = 4x$  in XY- plane from (0,0) to (4,4).
11. Evaluate  $\iint_R xy dx dy$  when  $R = [2,5; 1,2]$
12. Evaluate  $\iint_R \frac{dx dy}{\sqrt{1-x^2}\sqrt{1-y^2}}$  when  $R = [0, 1; 0, 1]$
13. Evaluate  $\iint_R \sqrt{4x^2 - y^2}$  when R is a triangle bounded by the lines  $y=0$ ,  $y=x$  and  $x=1$ .

### SECTION - B

14. Find the directional derivative of  $\phi = xy + yz + zx$  at A in the direction of  $\overline{AB}$ , where

$$A = (1, 2, -1), \quad B = (-1, 2, 3)$$

15. Prove that  $\text{div. Curl } \vec{f} = 0$

16. If  $\vec{F} = y\vec{i} + z\vec{j} + x\vec{k}$ , find the circulation of  $F$  round the curve,  $C$  where  $C$  is the

$$\text{Circle } x^2 + y^2 = 1, z = 0.$$

17. If  $F = 3xy\vec{i} - 5z\vec{j} + 10x\vec{k}$  evaluate  $\int \vec{F} \cdot d\vec{s}$  along  $x = t^2, y = 2t^2, z = t^3$  from  $t = 1$  to  $t = 2$ .

18. Evaluate  $\int_V F dV$  when  $F = x\vec{i} + y\vec{j} + z\vec{k}$  and  $V$  is the region bounded by

$$x=0, y=0, y=6, z=4 \text{ and } z=x^2.$$

### PART - III

Answer any **FOUR** questions from the following choosing at least **ON** question from each section. Each question carries 8 marks. 4X8M=32M

### SECTION - C

19. Between  $(0,0)$  and  $(a,2a)$  evaluate  $\int_C (x^2 + y^2) dx$  and  $\int_C (x^2 + y^2) dy$  where  $C$  is the arc of the parabola  $y^2 = 4ax$

20. Sketch the region of integration and write an equivalent double integral with order of Integration reversed and evaluate it  $\iint 3y dx dy$ .

21. In the integral  $\iint (4-y) dx dy$ , change the order of integration, and evaluate the integral.

### SECTION - D

22. Prove that  $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$

23. If  $F = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$  find  $\int_S F \cdot N ds$  by divergence theorem where  $S$  is surface of

the cube bounded by  $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ .

24. State and Prove Stokes theorem.

**P.R.GOV.T.COLLEGE (AUTONOMOUS), KAKINADA**

**III B.Sc. MATHEMATICS, Semester VI (w.e.f 2016-2017)**

**Course Code: Multiple Integrals & Vector Calculus**

**Total Hrs. of Laboratory Exercises: 45 @ 3 hr / Week in 15 Sessions**

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**Suggested topics for Problem Solving Sessions**

1. Line Integral-I
2. Line Integral - II
3. Double Integral-I
4. Double Integral-II
5. Directional Derivatives and Directional Derivative of Vector Point Function
6. Differential Operators
7. Integration of Vectors
8. Integral Transforms

**Problem Solving Sessions Examinations Pattern**

**End of the VI semester**

**(Course: Multiple Integrals & Vector Calculus)**

**PRACTICAL EXAMINATION FOR VI Semester : 50 Marks**

Written examination : 25 M

Record : 10 M

Viva- voce : 05 M

Cont.Ass. : 10 M

**TOTAL 50 M**